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Neutrino Mixing Matrix and Leptogenesis from Quantum Gravity

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Abstract We assume that the neutrino masses and mixing arise through physics at a scale intermediate between Planck Scale and the electroweak scale. Quantum gravitational (Planck scale) effects lead to an effective $SU(2)_L \times U(1)$ invariant dimension-5 Lagrangian involving neutrino and Higgs fields, which gives rise to additional terms in neutrino mass matrix. There additional term can be considered to be perturbation of the GUT scale bi-maximal neutrino mass matrix. We assume that the gravitational interaction is flavor blind. In this paper, we point out that due to Planck scale effects, in exact bimaximal model of neutrino mass matrix, measure $J_{\mathrm{CP}}^{\max} \neq 0$, it is possible to have lepton asymmetry of the universe.

Keywords Neutrino mixing matrix · Leptogenesis · Planck scale

1 Introduction

The current neutrino experiments has provided a strong evidence for non zero masses and mixing [1–5]. It is one of the direct indication for new physics beyond the Standard Model. An interesting point to understand the origin of the neutrino masses and mixings. An important question of interest related to the neutrino mass is leptogenesis [6, 7]. The most promising mechanism for generating a baryon asymmetry of the universe is through lepton asymmetry of the universe at the lepton number violating scale. There has been several attempts to understand the origin of the quark masses and mixing. In this article, we shall study the mixing matrix and possibility of leptogenesis. The neutrino masses could originate from either triplet Higgs field [7] or see-saw mechanism [8]. The question of CP violation in the lepton sector, we shall start with the parametrization of the CKM matrix in the quark

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sector

$$U_v = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{23}s_{13} \end{pmatrix}, \tag{1}$$

where, $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$ (labels $i, j = 1, 2, 3$) θ_{ij} and δ are the mixing angles and CP phase present in the mixing matrix present in leptons sector. The strength of CP violation in lepton sector related to the Jarlskog Invariant [9, 10]. J is the standard mixing parametrization is given by

$$\begin{aligned} J &= \mathrm{Im}\left(U_{e1}U_{e2}^{*}U_{\mu 1}^{*}U_{\mu 2}\right), \\ &= \frac{1}{8}\sin 2\theta_{12}\sin 2\theta_{23}\sin 2\theta_{13}\cos\theta_{13}\sin\delta. \end{aligned} \tag{2}$$

All the source of CP violation has to be related to this quantity J. If we predict J in any model, we can say existence of CP violation in the model. For example to generate lepton asymmetry of the universe one require CP violation, $\delta \neq 0$, if there is no CP violation in any model and hence leptogenesis will not possible. Consider the bimaximal neutrino mixing, in which $\theta_{13} = 0$. In this case, $J = 0$, implying that there is no CP violation in lepton sector and hence leptogenesis is impossible in any model which produces exact bimaximal neutrino mixing matrix. In Sect. 2, we outline the neutrino mixing due to Planck scale effects. In Sect. 3, numerical results and Sect. 4 is devoted to the conclusions.

2 Neutrino Oscillation Parameter Due to Planck Scale Effects

The neutrino mass matrix is assumed to be generated by the see saw mechanism [11–13]. We assume that the dominant part of neutrino mass matrix arise due to GUT scale operators and the lead to bi-maximal mixing. The effective gravitational interaction of neutrino with Higgs field can be expressed as $SU(2)_L \times U(1)$ invariant dimension-5 operator [14],

$$L_{\mathrm{grav}} = \frac{\lambda_{\alpha\beta}}{M_{pl}}(\psi_{A\alpha}\epsilon\psi_C)C_{ab}^{-1}(\psi_{B\beta}\epsilon_{BD}\psi_D) + h.c. \tag{3}$$

Here and every where we use Greek indices α, β for the flavor states and Latin indices i, j, k for the mass states. In the above equation $\psi_\alpha = (\nu_\alpha, l_\alpha)$ is the lepton doublet, $\phi = (\phi^+, \phi^\circ)$ is the Higgs doublet and $M_{pl} = 1.2 \times 10^{19}$ GeV is the Planck mass λ is a 3×3 matrix in a flavor space with each elements $O(1)$. The Lorentz indices $a, b = 1, 2, 3, 4$ are contracted with the charge conjugation matrix C and the $SU(2)_L$ isospin indices $A, B, C, D = 1, 2$ are contracted with $\epsilon = i\sigma_2$, σ_m $(m = 1, 2, 3)$ are the Pauli matrices. After spontaneous electroweak symmetry breaking the Lagrangian in (3) generated additional term of neutrino mass matrix

$$L_{\mathrm{mass}} = \frac{v^2}{M_{pl}}\lambda_{\alpha\beta}\nu_\alpha C^{-1}\nu_\beta, \tag{4}$$

where $v = 174$ GeV is the VEV of electroweak symmetric breaking. We assume that the gravitational interaction is "flavor blind" that is $\lambda_{\alpha\beta}$ is independent of α, β indices. Thus the

Planck scale contribution to the neutrino mass matrix is

$$\mu\lambda = \mu \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \tag{5}$$

where the scale μ is

$$\mu = \frac{v^2}{M_{pl}} = 2.5 \times 10^{-6}\ \text{eV}. \tag{6}$$

We take (5) as perturbation to the main part of the neutrino mass matrix, that is generated by GUT dynamics. To calculate the effects of perturbation on neutrino observables. The calculation developed in an earlier paper [13]. A natural assumption is that unperturbed (0^{th} order mass matrix) M is given by

$$\mathbf{M} = U^* \text{diag}(M_i) U^\dagger, \tag{7}$$

where, $U_{\alpha i}$ is the usual mixing matrix and M_i, the neutrino masses is generated by Grand unified theory. Most of the parameter related to neutrino oscillation are known, the major expectation is given by the mixing elements U_{e3}. We adopt the usual parametrization.

$$\frac{|U_{e2}|}{|U_{e1}|} = \tan\theta_{12}, \tag{8}$$

$$\frac{|U_{\mu 3}|}{|U_{\tau 3}|} = \tan\theta_{23}, \tag{9}$$

$$|U_{e3}| = \sin\theta_{13}. \tag{10}$$

In term of the above mixing angles, the mixing matrix is

$$U = \text{diag}(e^{if1}, e^{if2}, e^{if3}) R(\theta_{23}) \Delta R(\theta_{13}) \Delta^* R(\theta_{12}) \text{diag}(e^{ia1}, e^{ia2}, 1). \tag{11}$$

The matrix $\Delta = \text{diag}(e^{\frac{1\delta}{2}}, 1, e^{\frac{-i\delta}{2}})$ contains the Dirac phase. This leads to CP violation in neutrino oscillation $a1$ and $a2$ are the so called Majoring phase, which effects the neutrino less double beta decay. $f1$, $f2$ and $f3$ are usually absorbed as a part of the definition of the charge lepton field. Planck scale effects will add other contribution to the mass matrix that gives the new mixing matrix can be written as [13]

$$U' = U(1 + i\delta\theta),$$

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} + i \begin{pmatrix} U_{e2}\delta\theta_{12}^* + U_{e3}\delta\theta_{23,}^* & U_{e1}\delta\theta_{12} + U_{e3}\delta\theta_{23}^*, & U_{e1}\delta\theta_{13} + U_{e3}\delta\theta_{23}^* \\ U_{\mu 2}\delta\theta_{12}^* + U_{\mu 3}\delta\theta_{23,}^* & U_{\mu 1}\delta\theta_{12} + U_{\mu 3}\delta\theta_{23}^*, & U_{\mu 1}\delta\theta_{13} + U_{\mu 3}\delta\theta_{23}^* \\ U_{\tau 2}\delta\theta_{12}^* + U_{\tau 3}\delta\theta_{23}^*, & U_{\tau 1}\delta\theta_{12} + U_{\tau 3}\delta\theta_{23}^*, & U_{\tau 1}\delta\theta_{13} + U_{\tau 3}\delta\theta_{23}^* \end{pmatrix}. \tag{12}$$

Where $\delta\theta$ is a hermit ion matrix that is first order in μ [15, 16]. The first order mass square difference $\Delta M_{ij}^2 = M_i^2 - M_j^2$, get modified [15, 16] as

$$\Delta M_{ij}'^2 = \Delta M_{ij}^2 + 2(M_i \mathrm{Re}(m_{ii}) - M_j \mathrm{Re}(m_{jj})), \tag{13}$$

where

$$m = \mu U^t \lambda U,$$

$$\mu = \frac{v^2}{M_{pl}} = 2.5 \times 10^{-6}\ \text{eV}.$$

The change in the elements of the mixing matrix, which we parametrized by $\delta\theta$ [15, 16], is given by

$$\delta\theta_{ij} = \frac{i\mathrm{Re}(m_{jj})(M_i + M_j) - \mathrm{Im}(m_{jj})(M_i - M_j)}{\Delta M_{ij}'^2}. \tag{14}$$

The above equation determine only the off diagonal elements of matrix $\delta\theta_{ij}$. The diagonal element of $\delta\theta_{ij}$ can be set to zero by phase invariance. Using (12), we can calculate neutrino mixing angle due to Planck scale effects,

$$\frac{|U_{e2}'|}{|U_{e1}'|} = \tan\theta_{12}', \tag{15}$$

$$\frac{|U_{\mu 3}'|}{|U_{\tau 3}'|} = \tan\theta_{23}', \tag{16}$$

$$|U_{e3}'| = \sin\theta_{13}'. \tag{17}$$

For degenerate neutrinos, $M_3 - M_1 \cong M_3 - M_2 \gg M_2 - M_1$, because $\Delta_{31} \cong \Delta_{32} \gg \Delta_{21}$. Thus, from the above set of equations, we see that U_{e1}' and U_{e2}' are much larger than U_{e3}', $U_{\mu 3}'$ and $U_{\tau 3}'$. Hence we can expect much larger change in θ_{12} compared to θ_{13} and θ_{23} [16]. As one can see from the above expression of mixing angle due to Planck scale effects, depends on new contribution of mixing matrix $U' = U(1 + i\delta\theta)$. The above statements are not dependent on the exact from of the matrix λ given in (5). They hold true for any form of λ, as long as all its elements are of order 1. New contribution of mixing matrix also changes the electromagnetic properties and CP symmetry of neutrinos [20–22].

3 Numerical Results

We expect the mixing angles from GUT scale operators to be determine by some symmetry. We assume that, just above the electroweak breaking scale, the neutrino masses are nearly degenerate and the mixing are bi maximal, with the value of the mixing angle as $\theta_{12} = \pi/4, \theta_{23} = \pi/4$ and $\theta_{13} = 0$. Taking the common degenerate neutrino mass to be 2 eV, which is the upper limit coming from tritium beta decay [17]. We compute the modified mixing angles using (15) to (17). We have taken $\Delta_{31} = 0.002\ \text{eV}^2$ [18] and $\Delta_{21} = 0.00008\ \text{eV}^2$ [19]. For simplicity we have set the charge lepton phases $f_1 = f_2 = f_3 = 0$. Since we have set the $\theta_{13} = 0$, the Dirac phase δ drops out of the zeroth order mixing angle. In Table 1, we list the modified neutrino mixing angles for some sample value of $a1$ and $a2$. Due to Planck scale

Table 1 Maximum value of J_{CP}^{max} and modified mixing angles for some sample of a_1 and a_2. Input value are $\Delta_{31} = 0.002$ eV^2, $\theta_{12} = 45°$, $\theta_{23} = 45°$, $\theta_{13} = 0°$

$a1$	$a2$	θ'_{12} in degrees	θ'_{23} in degrees	θ'_{13} degrees	J_{CP}^{max}
0°	0°	48.57	45.00	0.28	0.0012
0°	45°	46.87	44.93	0.22	0.0009
0°	90°	44.99	44.86	0.13	0.0005
0°	135°	46.94	44.93	0.22	0.0009
0°	180°	48.57	45.00	0.28	0.0008
45°	0°	46.68	45.07	0.22	0.0003
45°	45°	44.96	45.00	0.20	0.0006
45°	90°	43.09	45.93	0.09	0.0009
45°	135°	45.03	45.00	0.14	0.0009
45°	180°	46.68	44.07	0.22	0.0006
90°	0°	45.00	45.14	0.14	0.0004
90°	45°	43.28	45.07	0.10	2.16×10^{-7}
90°	90°	41.42	45.99	0.00005	0.0004
90°	135°	43.00	45.07	0.10	0.0006
90°	180°	45.00	44.14	0.14	0.0009
135°	0°	46.68	45.07	0.22	0.0006
135°	45°	44.96	45.00	0.14	0.0009
135°	90°	43.09	44.93	0.09	0.0006
135°	135°	45.03	45.00	0.20	0.0003
135°	180°	46.68	45.07	0.22	0.0008
135°	0°	48.57	45.93	0.22	0.0009
135°	45°	46.87	44.93	0.13	0.0009
135°	90°	44.99	44.86	0.22	0.0005
135°	135°	46.94	44.93	0.22	0.0009
135°	180°	48.57	45.00	0.28	0.0012

effects, only θ_{12} have reasonable deviation and θ_{23}, θ_{13} deviation is very small less then 0.3° [16]. From Table 1, we can see for bimaximal mixing pattern $\theta_{12} = \pi/4, \theta_{23} = \pi/4$, and $\theta_{13} = 0$. We get the non zero value of modified mixing angle $\theta'_{13} < 0.3°$, gives non zero value. If $\theta_{13} \neq 0$ due to Planck scale effects, we can extracted the possible CAP violation. Due to Planck scale effects, non zero value of mixing angle θ_{13} indicate the possible CP violation in neutrino sector.

From Table 1 one may consider a deviation from the exact bimaximal neutrino mixing matrix and gives $\theta_{13} \neq 0$, indicates the CP violation. Since the CP phase δ is an independent parameter, with our present knowledge of mixing matrix U_v. We can compute the maximum J_{cp}^m, by choosing $\delta = \frac{\Pi}{2}$. We obtained J_{cp}^m, at various non zero value of θ'_{13}. The result are shown in Table 1, from the Table 1, we can put a limit on the amount of CP violating from the non zero value of θ'_{13},

$$J_{CP}^m = 0.0012. \tag{18}$$

4 Conclusions

We assume that the main part of neutrino masses and mixing from GUT scale operator. We considered these to be 0^{th} order quantities. The gravitational interaction of lepton field with SM Higgs field give rise to a $SU(2)_L \times U(1)$ invariant dimension-5 effective Lagrangian give originally by Weinberg [14]. On electroweak symmetry breaking this operators leads to additional mass terms. We considered these to be perturbation of GUT scale mass terms. We compute the first order correction to neutrino mass eigen value and mixing angles. In [16], it was shown that the change in θ_{13}, θ_{23} is very small (less then $0.3°$) but the change in θ_{12} can be substantial about $\pm 3°$ and changes in modified mass square difference $\pm(1.0 + 0.5) \times 10^{-5}$ eV^2 [23]. The change in all the mixing angle are proportional to the neutrino mass eigenvalues. To maximizer the change, we assumed degenerate neutrino mass 2.0 eV. For degenerate neutrino masses, the change in θ_{13}, θ_{23} are inversely proportional to Δ_{21}. Since $\Delta_{31} \cong \Delta_{32} \gg \Delta_{21}$, the change in θ_{12} is much larger than the change in other mixing angle. To summarize, we have shown that, due to Planck scale effects, it is possible to estimate the maximum allowed value of the Jarlskog invariant measure of CP violation. Sine the CP violation is an independent parameter, one can assume a maximum value of unity for this quantity to measure the Jarlskog invariant, CP violating parameter. In bimaximal mixing model of neutrino mass matrix, this measure $J_{\text{CP}}^{\max}$ vanish implying no CP violating. Due to quantum gravity (Planck scale) effects gives non zero value of $J_{\text{CP}}^{\max} \neq 0$, it is possible to have lepton asymmetry of the universe.

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